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PB-258 036

Atmospheric Dispersion Parameters in Gaussian Plume Modeling. Part II

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Jun 76

280261

PB 258 036

EPA-600/4-76-036b
June 1976

Environmental Monitoring Series

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TECHNICAL REPORT DATA

1 REPORT NO. EPA-600/4-76-030b	2
4 TITLE AND SUBTITLE ATMOSPHERIC DISPERSION PARAMETERS IN GAUSSIAN PLUME MODELING: Part II. Possible Requirements for Change in the Turner Workbook Values	3 RECIPIENT'S ACCESSION NO.
	5 REPORT DATE June 1976
	6 PERFORMING ORGANIZATION CODE
7 AUTHOR(S) F. Pasquill	8 PERFORMING ORGANIZATION REPORT NO.
9 PERFORMING ORGANIZATION NAME AND ADDRESS Meteorology and Assessment Division Environmental Sciences Research Laboratory Research Triangle Park, North Carolina 27711	10 PROGRAM ELEMENT NO. 1AA009
	11 CONTRACT/GRANT NO.
12 SPONSORING AGENCY NAME AND ADDRESS Environmental Sciences Research Laboratory Office of Research and Development U.S. Environmental Protection Agency Research Triangle Park, North Carolina 27711	13 TYPE OF REPORT AND PERIOD COVERED In-house; Jan - Mar 1976
	14 SPONSORING AGENCY CODE EPA-ORD

15 SUPPLEMENTARY NOTES

16 ABSTRACT

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17 KEY WORDS AND DOCUMENT ANALYSIS

a DESCRIPTORS	b IDENTIFIERS/OPEN ENDED TERMS	c COSATI Field/Group
Air pollution *Atmospheric diffusion *Wind (meteorology) *Plumes *Mathematical models	Gaussian plume	13B 04A 04B 21B 12A

18 DISTRIBUTION STATEMENT RELEASE TO PUBLIC	19 SECURITY CLASS (This Report) UNCLASSIFIED 20 SECURITY CLASS (This page) UNCLASSIFIED	21 NO. OF PAGES 32 22 PRICE $4.50 - 3.00

EPA Form 2220-1 (9-73)

EPA-600/4-76-030b
June 1976

ATMOSPHERIC DISPERSION PARAMETERS IN GAUSSIAN PLUME MODELING
Part II. Possible Requirements for Change in the Turner Workbook Values

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PREFACE

The increasing concern of the last decade in environmental issues, and the fuller appreciation that air quality simulation modeling may provide a unique basis for the objective management of air quality, has generated an unprecedented interest in the development of techniques for relating air quality and pollutant emissions through appropriate modeling of the atmospheric transport and dispersion processes that are involved. A multitude of recent publications in the U.S.A. and elsewhere points to the wide interest at many different levels of local, state, regional and national planning, and testifies to the widespread acceptance of meteorological-type air quality modeling as an important rational basis for air quality management. This point of view is now internationally recognized in most industrial countries.

Earlier attitudes towards the quantitative estimation of atmospheric dispersion of windborne material from industrial and other sources were strongly influenced by a system introduced in 1958, and published in 1961, by Dr. F. Pasquill of the Meteorological Office, United Kingdom. This was followed in 1962 by the publication of his definitive textbook on "Atmospheric Diffusion," which includes detailed consideration of the well-known simple "Gaussian-plume" model for the average concentration distribution in space from an elevated continuous point-source under steady conditions. The unique feature, however, of the Pasquill system is the method by which the critical parameters expressing the downwind

spread of the plume might be estimated in terms of the ambient meteorological conditions. These estimates were later expressed in slightly more convenient, although exactly equivalent form, by Dr. F. Gifford, and this so-called Pasquill-Gifford system for dispersion estimates has been widely used ever since. It was early given some general endorsement as a valuable practical scheme by the Public Health Service of the U.S. Department of Health, Education, and Welfare, by the publication in 1967 of the "Workbook of Atmospheric Dispersion Estimates" (Public Health Service Publication No. 999-AP-26) by D. B. Turner, that exclusively utilized this system of Gaussian-plume dispersion parameters.

In spite of the gradual appearance in recent years of air quality models based on more sophisticated formulations of the atmospheric processes, e.g., through fluid-dynamical equations assumed to govern the physical processes of transport and turbulent diffusion, great use continues to be made of the simpler Gaussian-plume models. However, a direct consequence of the unprecedented interest in the subject has been the publication of many attempts to confirm or improve the realism of the dispersion estimates. These and other matters relating to the substantial progress made in recent years in understanding atmospheric dispersion, are discussed in a much revised 2nd Edition of the Pasquill book, that was published late in 1974. Under these circumstances it seemed desirable to examine critically the possible requirements for change in the Turner Workbook values for dispersion, that have been so widely used since 1967. The present two-part

report was prepared to meet this need.

It was extremely fortunate that Dr. Pasquill was available for detailed discussions during its preparation (1975-76), both while he was a Visiting Professor at the North Carolina State University and also the Pennsylvania State University (under research grant support of the EPA), and also as a Visiting Scientist to the Meteorology and Assessment Division of the E.P.A., Research Triangle Park, N.C. Even more fortunate has been Dr. Pasquill's willingness to assume responsibility for preparation of the second part of the report, which critically examines the possible requirements for change of the Turner Workbook values. The first part was prepared by Dr. Allen Weber of the Department of Geosciences at North Carolina State University, in consultation with Dr. Pasquill and the undersigned. It provides a reasonably comprehensive review of current systems and possible future developments, and is a necessary input for the critical examination of the second part. It is perhaps unnecessary to emphasize that there are still many problems of dispersion that are unlikely to be resolved satisfactorily in terms of simple Gaussian-plume models. However, it is hoped that the present publication will provide a more up-to-date basis for continuing the successful treatment of the many important practical problems that can be analysed by this simple approach.

Research Triangle Park,
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March 1976

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ABSTRACT

The basis of the original Pasquill-Gifford curves used in the Turner Workbook is restated and consideration given to those features of the curves which are now regarded as especially questionable.

Data on crosswind spread from various field tests are reviewed to emphasize the useful working relation which holds between σ_y and the standard deviation of the wind direction fluctuation. Some new trial calculations of vertical spread are carried out in the light of recent work using the gradient-transfer approach, recent similarity analyses, new observational data on the structure of turbulence in the convective boundary layer, and Deardorff's modeling of the mixed layer.

Recommendations are made concerning the use of wind direction fluctuation data for estimating σ_y, for various adjustments and constraints to be applied as an interim measure to the existing σ_z curves, and for continuing work required in the progress toward a final revision of the Workbook.

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ACKNOWLEDGEMENTS

The writer wishes to thank Dr. G. A. Briggs, Mr. F. I. Gifford, Mr. D. B. Turner and Dr. A. H. Weber for helpful discussions during the preparation of the review, and Drs. J. C. Kaimal and J. J. Caughey for access to draft data summaries from the Minnesota 1973 atmospheric boundary layer study.

SECTION 1

INTRODUCTION

As pointed out in the preface to this two-part report the dispersion parameters σ_y and σ_z adopted in the Workbook[1*] are known widely as 'Pasquill-Gifford' curves. In essence, they are estimates originally offered tentatively by the writer, in unpublished form in the Meteorological Office in 1958 and in published form in 1961.[2]

The purpose of this second part of the report is to provide an appraisal, against the background of subsequent dispersion research and of the review in Part I, of the requirements for change in the Workbook curves (Figs. 3.2 and 3.3 in that publication). It was thought appropriate to begin the appraisal with a brief restatement of the basis and scope of the original estimates, emphasizing certain points which may sometimes be overlooked and also certain points which in restrospect, need clarification. Consideration of the various aspects in which the estimates are unacceptable or questionable then follows in Section 3. It is to be noted at the outset that completion of the appraisal to the point of a definitive full revision of the Workbook values is not immediately possible.

There are specific points, including the consolidation of the relation between σ_y and wind direction fluctuation and the explicit representation of the effect of surface roughness on dispersion, in respect of which immediate recommendations for change appear fully justified.

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*To facilitate frequent reference to certain publications the numeral system of referencing has been adopted.

these are presented in Section 4. There are, however, features in respect of which further time must be allowed for a duly considered incorporation of currently emerging material. This appears to be particularly the case for vertical spread in very unstable conditions, which, in view of the current state of development, is one of the topics given most attention in Section 3 and Appendix B. Finally, and not surprisingly, there still remain features which await theoretical clarification and observational confirmation. The developments which accordingly appear to be essential for progress to a more complete revision of the 'PG' curves are briefly noted in Section 5.

SECTION 2

RESTATEMENT OF THE BASIS AND SCOPE OF THE ORIGINAL 'PASQUILL-GIFFORD' CURVES

The formulation of the PG curves was stimulated by increasing demands for methods of estimating dispersion which were both simple to apply in terms of the idealized 'Gaussian plume' description of the spread of a continuous point source plume, and which could be applied in terms of routinely available meteorological data. Even so, the view was adopted that the best prospect for generality and reliability in the estimates lay with a maximum use of the theoretical principles of turbulent diffusion and a minimum dependence on uncritical analyses of pollution surveys.

In the latter respect it is perhaps appropriate to emphasize that the estimates were not just a compilation of observational experience on dispersion *per se*. They did embody some *a priori* quality in two notable respects. First, the attempted generalization about σ_y was considered as far as possible in terms of a simple adaptation of the Taylor statistical theory. Indeed a preference was clearly stated for a direct use of wind direction fluctuation data in estimating σ_y, and the σ_y estimates contained in the PG curves were provided for use in the absence of such data. Inevitably, these estimates are of limited value, derived as they were from preliminary statistics of wind direction fluctuation and applicable to a particular (3 min) sampling time. A second *a priori* element, embodied in the σ_z curves, is the gradient transfer theory result for short distance of travel and neutral conditions of flow. Demonstration of the correctness of this result, in terms of data on the wind profile

over a natural rough surface, had been provided by Calder.[3]

For extension of the estimates beyond the two fundamental 'reference points' of the preceding paragraph it was, of course, necessary to appeal to a subjective interpretation of the limited experience then available on rates of dispersion in the atmosphere. Note that the σ_z estimates referred strictly to a surface release of material (i.e. $H = 0$ or in more general terms effectively when $H \ll \sigma_z$). However, in the absence of strong evidence or argument to the contrary, acceptance of the estimates even for elevated releases (in more general terms for $H \gtrsim \sigma_z$) was encouraged.

Although the relevance of sampling time was more specifically clear for σ_y properties, there was an implication which was not made clear originally concerning σ_z. For a release which is effectively at the surface (i.e. $H \ll \sigma_z$) it is logically necessary to expect the σ_z property to become independent of sampling time t for $t \geq t_m$. This limiting sampling time t_m is that which is required for the vertical distribution to respond to the whole spectrum of the vertical component of turbulence. Experience on the latter property suggests that t_m will at first increase nearly linearly with t and then more slowly. As a rough guide it is suggested that the estimates of σ_z should be taken to apply for $t \geq t_m$ with $t_n = 10$ min for $H \leq 100$m, and $t_m = H/10$ min for H (in metres) ≥ 100. (note that the relation between σ_y and sampling time is also fundamentally tied to the spectrum of the y-component in this case but here the existence of a clear-cut short limit to t, beyond which σ_y no longer increases, is

generally precluded by the highly variable low-frequency content of the v-spectrum). Again, returning to σ_y, for sampling times less than t_m one cannot however expect a systematic increase of σ_y with t to be reflected in the ground-level concentration from an effectively elevated source, i.e., for $\sigma_y \leq H$. One must then expect an intermittency in the crosswind-integrated concentration at ground level, with ensemble-average values approximating to that for $t \geq t_m$.

In conclusion of this section, and for future convenient reference, the essential points of the background to the P6 curves are summarized in table 1.

[redacted]

SECTION 3

CONSIDERATION OF SOME SPECIALLY QUESTIONABLE FEATURES OF THE PG ESTIMATES

CROSSWIND SPREAD

Several recent discussions of the application of the Taylor statistical theory to practical relations between crosswind spread and wind direction fluctuation are available (Ref. 6, p 185 et seq., and Ref. 7 and 8). By way of further demonstration of the usefulness of the approach, some of the existing test data on σ_y are presented in rearranged form in Appendix A. Taken together with the previously published discussions, the following conclusions may be drawn:

(i) Irrespective of sampling time (within the practical range of a few minutes to about 1 hour), and irrespective of surface roughness and stability, there is a rough conformity to a 'universal' relation between σ_y/σ_θ and distance x, when the standard deviation σ_θ of the wind direction is evaluated over the sampling time, for distances of travel up to about 10 km.

(ii) At the shortest distances of interest (<0.1 km), the limiting form $\sigma_y = \sigma_\theta x$ is a reasonably good approximation on average (σ_θ in radians)

(iii) In general, we may write $\sigma_y = f(x)\, \sigma_\theta x$ on average, with $f(x)$ taking the values listed in Appendix A.

(iv) for individual samples the departures from the foregoing average relations are mostly within a factor of about 1.5 at short range and about 2.0 at long range.

(In all the foregoing relations note that for generality, irrespective of the magnitude of σ_θ, the quantity σ_y should be taken as referring to the distribution along an arc of radius x.)

The significance of these results to the PG σ_y curves is obvious. As recalled previously, these curves are for a sampling time of 3 min and a rather smooth surface. Over a much rougher surface, such as an urban area, the magnitude of σ_θ (and hence that of σ_y) will be increased, especially for shorter sampling times and strong winds, when the generation of the v-fluctuation is predominantly mechanical. In the latter case, the increase may be as much as a factor of two. For longer sampling times there may also be a substantial increase. Some examples of σ_y over a rather smooth site (Ref. 6, p 69 and Ref. 7) show increases by a factor of two for an increase of sampling time from 5 min to 1 hr.

The possible consequences of using the PG σ_y curves for calculation of concentrations for comparison with data obtained with much longer sampling times should be noted. It follows, from the systematic growth of σ_y with sampling time, that the frequency distribution of concentration experienced at a particular receptor from a particular

point source will have fewer very small concentrations (i.e., receptor outside the plume) and fewer very large concentrations (which arise partly from very narrow plumes) for a long sampling time than for a short sampling time. This may well partly explain the difference in calculated and observed frequency distributions reported by Klug (Ref. 16), where the observations were for 1-hr samples, if, as appears to be the case, the calculations were based on the PG σ_y curves as they stand.

Further demonstration of the direct importance of the actual wind direction variation is contained in the Idaho A.R.L. study (Ref. 13) already referred to in Appendix I. Here, the observed arc distribution of concentration from a continuous point source (at radii up to 400 m) was compared with calculated distributions. Closest agreement was achieved by subdividing the total sampling duration (1 hr) into 3-min sections, for each of which the plume spread was taken in accordance with the PG curves, and then superimposing the 3-min distributions on the arc at positions in accordance with the 3-min average wind directions.

The incorporation in the Workbook procedure of estimates of σ_y based on a more comprehensive knowledge of wind direction fluctuation should now be given careful consideration. It is necessary first to recall the point recognized in the analysis of Appendix I, namely, that σ_θ is not strictly a constant with respect

to height. In a well-mixed layer the expertence is that σ_v *per se* does not change markedly with height, though in stably stratified flow a decrease with height is to be expected, especially in the relatively high-frequency part of the spectrum. Noting that $\theta_v = \sigma_v/u$, and taking into account the usual increase of wind with height, it follows that σ_θ *per se* must be expected to decrease with height at a rate depending on the stability. In practice, therefore, it would seem to be necessary to consider specification of σ_θ at a reference height which should be increased in accordance with the progressive *vertical* spread of the plume.

Regarding the need argued above for an effective value $(\sigma_\theta)_e$ at some appropriate reference height, which might most logically be set equal to σ_z, there is, however, an interesting simplification which might be argued as follows. The value ultimately adopted for σ_y is to be used in the expression for time-mean concentration $\bar{c}(x)$, and it will be recalled that in the denominator of that expression we have the product $\bar{u}\ \sigma_y$, in which the mean wind speed $\bar{u}$ also should be an effective value in relation to the vertical spread of the plume, i.e., we should use $\bar{u}_e\ \sigma_y$. If there is no variation of σ_v with height, then to an approximation (with $\bar{t}$ the average time of travel of of 'particles' to distance x)

$$u_e \sigma_y = u_e \sigma_v \bar{t} = u_e \sigma_v x/u_e = \sigma_v x$$

i.e., the need for a specific reference height is cancelled out as far as the variation of $\bar{u}$ with height is concerned, though, of course, it remains insofar as σ_v changes with height.

Provision of appropriate data for σ_θ requires either real-time measurements or a climatology of the form utilized in the construction of the PG σ_y curves at short range, but now necessarily generalized in terms of sampling duration. In principle, such a climatology is derivable from the accumulated knowledge of the v-spectrum, though there are considerable uncertainties arising from the poorly-defined low-frequency form of that spectrum (e.g., see Ref 7). An interim approach which is worth considering in the present context is to consider the total σ_θ in two parts (additive in terms of σ^2) representing high-frequency and low-frequency contributions separately. The relatively-high-frequency part may be represented by a σ for a sampling duration of 1 min, say, for which we could continue to use the 'preliminary' climatology incorporated in the PG curves, until a better climatology is constructed. The low-frequency part (to be added in terms of σ^2) would require an estimate of σ for an averaging (smoothing) time of 1 min and for the total sampling duration of interest (usually 1 hr). An obvious convenience is that the latter type of statistic does not require rapid response records of wind direction and is derivable from routine slow-response records.

In terms of σ_y the suggestion in the previous paragraph would be equivalent to retaining the PG curves pro-tem and adding to those σ_y' a component given by

$$\left[\sigma_\theta^2\right]_{t,s}^{\frac{1}{2}} x^{\frac{1}{2}} f(x)^{\frac{1}{2}}$$

where the subscripts t and s refer to sampling duration and averaging (smoothing) time, the latter being 3 min in the present proposal, so giving a total σ_y^2.

Finally, there is the complication of the augmenting effect of wind shear to be considered (Ref 6, p 156, 229, 363). A rough rule which has been suggested in the last of the foregoing references is to add (to the total σ_y^2 of the preceding paragraph), from, say, 20 km onward, a third component of variance which is $0.03\Delta\theta^2x^2$ where $\Delta\theta$ is the turning of wind direction over the total depth of the plume, in radians. Note that this shear contribution is proportional to x^2, i.e., the corresponding σ is proportional to x, undoubtedly a very rough and tentative approximation.

THE EFFECT OF SURFACE ROUGHNESS ON VERTICAL SPREAD

The effect of surface roughness on vertical spread is represented basically through the friction velocity u_*. This parameter appears directly in the similarity approach (Ref. 6, p 118 et seq) and in the specification of the momentum-transfer form of the eddy diffusivity; it is implicitly involved in the properties of the vertical component

of eddy velocity. The r.m.s. value of the latter (σ_w) again appears in similarity considerations (see Appendix 2) and in the K formulation (see Ref. 6, p 105) used by F. B. Smith in deriving numerical solutions of the two-dimensional equation of diffusion.

Although the theoretical dependence of σ_z on surface roughness has not been comprehensively verified by observations of atmospheric dispersion, data for two widely different roughnesses, namely, open grassland and urban areas, with z_0 values respectively 3 and 100 cm, are broadly consistent with theory[15]. For travel distances of a few kilometres the corresponding values of σ_z are in the ratio 1 : 2 roughly. A more complete, though tentative, theoretical specification is contained in a nomogram (see Ref. 6, p 377) constructed by F. B. Smith from the numerical solutions referred to above. This specification is considered tentative, especially at distances of travel greater than a few kilometres, because of uncertainties remaining in the estimates of K in the upper part of the boundary layer and, especially in very unstable conditions when buoyancy forces become dominant over frictional forces, because the gradient-transfer assumption is then seriously questioned (see following section). With these qualifications, however, plausible corrections for different terrain roughnesses may now be made and should be applied to the PG σ_z curves (which at any rate for the shorter ranges, say, a few kilometres, are appropriate only to z_0 = 3cm, though at longer range the effects of somewhat larger roughness have been implied through incorporation of the experience of long range tracer studies).

VERTICAL SPREAD BEHAVIOUR IN UNSTABLE CONDITIONS

The PG curve for stability category A contains a pronounced monotonic increase of $d\sigma_z/dx$ with x. This shape was based on the experience provided by the 'Prairie Grass' dispersion study.[4] It should be recalled, however, that the study provided direct measurements of σ_z (i.e. values derived from an observed vertical distribution) only for a distance of travel of 100 m, and indirect estimates (i.e., from the reduction of ground-level concentration with distance) only up to 800 m downwind. For greater distances the PG curve is essentially a subjective extrapolation. The further evidence and argument which may now be brought to bear on the acceptability of this curve (and to a lesser degree on that of the B and C curves) is summarized below.

(i) The new and essentially theoretical derivations[5,6] of σ_z from numerical solutions of the two-dimensional diffusion equation using plausible profiles of K based on early-stage data on turbulence over the whole depth of the boundary layer (and not on dispersion data), yield a σ_z,x curve in unstable conditions which does show the property of $d\sigma_z/dx$ increasing with x, but only transiently. This increase does not continue beyond a σ_z of about 100 m.

(ii) Further new estimates of σ_z, on a basis equivalent to the foregoing numerical solutions, have been derived during the progress of the present review. These were obtained from a

convenient similarity representation of the foregoing numerical solutions[7,8], using new data on the w-spectrum in the atmospheric mixing layer, obtained in a joint U.S.A.F.C.R.L. and U.K. Meteorological Office boundary layer study.[9] It is to be emphasized that those new estimates of σ_z do not explicitly contain any contribution from mechanisms other than that of gradient-transfer.

(iii) Early results of the 2nd-order closure modelling developed by ARAP (Fig. 16 of Ref. 10) also show a section with $d\sigma_z^2/dx$ increasing with x. This curve is for a condition equated with Category C and was evaluated for a specified depth of mixing.

(iv) The existence of a regime with increasing $d\sigma_z/dx$ is predictable on similarity grounds for the special case of effectively-windless convection. In such conditions, postulating reasonably that the 'velocity' of vertical spread ($d\sigma_z^2/dt$) must, in the region sufficiently below any upper limiting boundary, be determined completely by two parameters, namely, a buoyancy parameter determined by the surface heat flux H_0, and the height z as the only relevant length scale, it follows on dimensional grounds that

$$d\sigma_z^2/dt \propto (gH_0z/\rho c_p T)^{1/3} \qquad (1)$$

On integrating

$$\sigma_z^2(t) \propto (gH_0/\rho c_p T)^{1/3} t^{2/3} \qquad (2)$$

this prediction of σ_z growing as $t^{1/2}$ has been ascribed to Taylor[11] and has been found by Deardorff and Willis (Fig. 3 of Ref. 11) to be substantially confirmed by laboratory-convection-tank data over the range $0.15 < \sigma_z/z_i < 0.4$, z_i being the total depth of the mixed layer. It is noteworthy that in Deardorff's considerations this relation emerges as a special regime in an overall 'mixed-layer' relation between σ_z/z_i and the dimensionless time t_* formed by w_*t/z_i, with $w_* = (gH_0z_i/\rho c_p T)^{1/3}$:

the simple relation

$$\sigma_z/z_i = a\, t_*^{1/2} \qquad (3)$$

with 'a' constant implies, in keeping with the definitions of t_* and w_*, a dependence of $d\sigma_z/dt$ on $z^{-1/3}$, but a complete lack of dependence on z_i. While this result does not seem surprising in one respect, i.e., that the vertical transport of particles is not affected by an upper boundary until some of the particles are nearing the boundary (when $\sigma_z/z_i = 0.4$) it is surprising, and indeed paradoxical, in another respect. The crucial point of Deardorff's approach is that the properties within a convectively mixed layer are uniquely related to the two parameters z_i and w_*. His data support this hypothesis in respect of both σ_w and σ_z, but with an important difference, namely, that for σ_w the influence of z_i becomes apparent at $z/z_i = 0.1$, whereas that on σ_z apparently

does not emerge strongly until z/z_i is nearly 0.5. (In the later stages of this review, however, it has been learned that a more recent presentation and discussion of laboratory data is in course of publication by Deardorff, and it has yet to be clearly settled how far the new results confirm or modify the foregoing statements.)

(v) In regard to the relation between σ_w/w_* and z/z_i, which on Deardorff's hypothesis is expected to be a universal function, it is noteworthy that the form demonstrated by Deardorff's[11] laboratory data is not completely confirmed by the atmospheric data recently acquired in Minnesota and referred to in (ii) above. There is agreement up to $z/z_i = 0.5$ (and up to 0.1 both support σ_w proportional to $z^{1/3}$ and independent of z_i), but for larger z/z_i the two sets of data diverge (see Fig. 1 of Ref. 11 and Fig. 18 of Ref. 9), the laboratory data showing a distinct fall in σ_w/w_* as the top of the layer is approached whereas the atmospheric data show, if anything, a very slight increase.

Details of new estimates of σ_v for unstable conditions, using both the gradient-transfer approach, and the free-convection law in Eq. 3, are set out in Appendix B. Three cases have been considered, the first referring to relatively smooth open country (to which the P8 curves apply), and the second and third to an urban area, for which estimates are available from the St. Louis tracer study.

Specific conclusions which emerge from these results are set out at the end of the appendix. It is clear that a closer approach to the extrapolated (long-range) section of the PG 'A' curve and to the St. Louis 'B' and 'E' curves has been achieved in terms of the 'free-convection' law and, implicitly, in terms of the Deardorff 'mixed-layer' law. Up to a point this is not surprising and it is clear that final revision of the PG curves must take into account the new insight into the effects of convective motion. However, because of the discrepancies and conflicts which have been noted in (iv) and (v) above and which have not been resolvable in the present review, and because the latest report on the Deardorff modelling has not been available to us in time for full consideration, it would certainly not be appropriate to attempt a final revision immediately.

Aside from the magnitude of σ_y there is another feature of the Deardorff modelling which is highly relevant to the prediction of the actual concentration levels in a convectively-mixed layer. The Deardorff and Willis data[11] on vertical dispersion from a low-level release show the appearance of an elevated maximum in the (effectively) crosswind integrated concentration, at the stage when $\sigma_y/z_i \simeq 0.25$. The level of this maximum rises and the surface concentration goes through a temporary stage in which it is less than the ultimate uniformly-mixed value. The full understanding of this striking result and confirmation of the extent of its applicability to the atmospheric mixed layer are important steps to be achieved.

OTHER ASPECTS

There are several features which have not been given extensive consideration during the preparation of this review, and these are noted below with brief comment and reference.

(i) Vertical Spread in Stable Flow

From the résumé given in Part I it is evident that estimates of σ_z in stable conditions vary over a wide range. In view of the expected (though not yet well-documented) decrease of σ_w with height a dependence on height of release H is also to be expected over the distance of travel for which $\sigma_z < H$. The BNL curves may be thought to contain some evidence of this dependence, as the 'stable' curve falls distinctly below the PG 'F' curve, despite the fact that it refers to a much rougher surface. However, the evidence is somewhat ambiguous, since the precise equivalence of the stability classifications in the two cases is in some doubt. This particular point, as well as the more general question of the dispersive character of stable flow, is in need of further critical examination. The obvious and preeminent requirement is continued progress in the description of the characteristic profiles of the intensity and scale of turbulence, in the distinction between turbulence and wave-motion, and in the ongoing work to which our attention has been drawn in the course of the present review (G. A. Briggs - private communication).

(ii) The Effect of Elevation of Release and Plume Rise on Dispersion

The possibly special importance of elevation of release in stable

conditions has already been mentioned, but as noted in Part I there is a more general problem arising from the inapplicability of either the simple statistical or gradient-transfer treatment in the crucial stage of dispersion at which σ_y is similar to H (see Ref. 8). This is one of the problems for which the 2nd-order closure modelling seems to be specially necessary.

For strongly buoyant plumes there is also the contribution to spread (both vertical and horizontal) associated with the entrainment induced by relative vertical motion of plume and surrounding air. As previously discussed (Ref. 6, p 271), the addition to the 'passive' dispersion represented in the PG curves may be especially important for σ_z in stable conditions. It is to be noted, however, that for maximum ground-level concentration per se the resultant effect may not often be important, since the major part of the dispersion to ground level may often depend predominantly on the 'passive' part of the process. The condition for maximum ground level is roughly represented by $\sigma_z = (H_s + \Delta H)/2^{1/2}$, where H_s is the stack height and ΔH the height of rise. The data on rising plumes suggest that the induced σ_z is roughly $\Delta H/3.5$ which obviously cannot be the major contribution to the required σ_z even when ΔH is very large compared with H_s.

(iii) Vertical Spread in an Evolving Mixed Layer

Allowance for the more or less abrupt termination of the upward vertical dispersion by an overhead stable layer is included in the

Workbook (see pp 7 and 36 thereof). This procedure needs to be generalized in terms of the typical evolution of the depth of the mixed layer in the surface heating cycle following nocturnal stabilization of the boundary layer, as discussed in Ref. 6, p 379. If the mixed layer is of depth h'(t) at time t, it is clear that the $\sigma_z(x)$ appropriate to a source at distance x upwind is in accordance with unbounded σ_z,x curves only up to a certain limit depending on h'. One simple rule may be stated immediately on the basis of the conventional 'image source' principle, for the case of the ground-level crosswind-integrated concentration (CWIC) from a ground-level source, with constant h'. The curve of CWIC against x is found to be adjusted quite sharply (for practical purposes effectively suddenly at the distance defined by $\sigma_z = 0.8h'$) from the vertically-unbounded form

$$CWIC = 2^{1/2}Q/\pi^{1/2}u\sigma_z, \quad 0 < \sigma_z \leq 0.8h', \tag{4}$$

to the 'uniform vertical distribution' form

$$CWIC = Q/uh', \quad \sigma_z > 0.8h', \tag{5}$$

For source-heights and receptor-heights other than zero, the curve is, of course, more complex, but nevertheless derivable on the same principle.

Section 4

SUGGESTIONS FOR INTERIM CHANGES IN THE WORKBOOK PARAMETERS AND PROCEDURE

Suggestions for the more satisfactory estimation of crosswind spread have already been considered at some length (see p. 7). For vertical spread (see p. 14), there are significant current developments which should be incorporated as soon as an adequate clarification and consolidation of the results have been achieved. In the meantime, it would be reasonable to retain the present Workbook values of σ_z with adjustments for roughness (in accordance with the discussion on p. 12), for induced spread of buoyant plumes (see p. 19), and for the effects of urban heating on lines which are briefly considered below.

Ideally, the σ_z curves for urban flow should be related to surface roughness and surface heat flux, with the latter representing the total effect of natural and man-made contributions, as was attempted in Appendix B for the unstable conditions. Until this can be satisfactorily accomplished, a reasonable simple procedure would be to adjust the PG curves for roughness as already discussed and then apply an additional correction for 'urban-heating' in accordance with the practical experience of the St. Louis study. This latter correction could be applied conveniently in the form of an 'adjusted (urban) stability category,' appropriately more unstable than the estimated 'open country' category. The experience suggests that this adjustment should be roughly 'half a stability category.'

The foregoing suggestions are collected in summary form in Table 2.

[redacted]

[redacted]

[redacted]

Notes on table 2

* this formula is strictly true only for small values (i.e., when tan $\vartheta \approx \vartheta$). For larger values (i.e., with wide and usually slow variation of wind direction) an appropriate procedure is to use the formula in a formula for concentration in which the y-distribution of concentration is replaced by an arc distribution at a radius r.

P for correct evaluation this σ_ϑ should be for an effectively infinitesimal averaging time (i.e., no loss of variance by 'smoothing' of the complete wind fluctuation). As an alternative one could continue to use the PG σ_y values, which refer to a sampling time of 3 min, and add to this value squared the square of the value obtained from the above formula with a σ_ϑ obtained from 3-min averages over the sampling time of interest.

SECTION 5

REQUIREMENTS FOR PROGRESS TOWARD A FINAL REVISION OF THE WORKBOOK PARAMETERS AND PROCEDURE

In view of the conflicts of evidence and gaps in knowledge which so obviously exist at present, by far the most important issue is the identification of the points at which new measurements and further critical analyses are most required. A short list of those which seem to the writer to be the most urgent is given below.

VERTICAL DISPERSION IN THE CONVECTIVELY MIXED LAYER

The present review has been a vivid reminder, if such were needed, of the very inadequate background of high-quality test data on the crucial features of vertical dispersion beyond the first kilometre of travel from a source. Improvement in this respect is vital, not only for resolution of the difficulties encountered here, but also as an ultimate essential test of the emerging 2nd-order closure treatments of dispersion, which treatments are seen by some as the main future hope of improved dispersion estimates. The necessity for this advanced type of treatment, especially in respect of convective transfer, has recently been emphasized by Lumley.[17]

The convective case is of specific practical importance in the familiar 'looping' and 'fumigation' aspects of power-plant plumes, and is of general importance as the condition for the 'upper bound' to a family of curves such as those of the PG system. Despite the various

attempts in the past to obtain definitive data on vertical spread there still remains a particular need for good measurements, say, by aircraft sampling of the plume from a surface release of passive tracer, of the progress to the stage of near-uniform distribution with height. This should be considered in the first instance over an ideal (Kansas or Minnesota-type) site, with supporting data on the surface heat flux, the mixing depth, and the wind profile. The experience of the Minnesota boundary layer study (Ref. 9) confirms that a useful study of the role of convective transfer is feasible even in moderate winds, although the preference should be for winds as light as possible consistent with predictability of the plume trajectory. The tracer sampling (crosswind traverses at several levels in the mixed layer) should be at intervals of downwind distance of, say, one mixing depth up to at least 5 mixing depths.

VERTICAL DISPERSION IN STABLE FLOW

The very stable condition represents the 'lower bound' to the family of σ_z curves and is, of course, the most serious condition for pollution from low-level sources. From the discussions which have been held during the present review there are signs that a concentration of effort might now be timely, especially in the respect of marshalling newly available turbulence data (or acquiring such data) and appraising the significance thereof in terms of effective eddy diffusivity. Whether an adequate background of dispersion data already exists in a form appropriate for testing of new ideas has yet to be seen, but should a need become

apparent for new data in this context, the observational requirement will be less of a problem than for the convective case, for the obvious reason that crucial data will be obtainable from sampling on masts or towers.

CRITICAL STUDY AND ANALYSIS OF 2ND-ORDER CLOSURE CALCULATIONS OF VERTICAL DISPERSION

The potential importance of the 2nd-order closure approach has already been noted in respect of convective mixing. Another important feature for which it could be the most profitable means of improved understanding is the case of the elevated source for the range over which $\sigma_z < h$. A review of the impossibility of properly representing the real physical action in terms of the simple statistical and gradient-transfer theories has recently been given elsewhere.[8] Acquisition of experience in the application of the 2nd-order technique to this particular difficulty could be most enlightening.

STABILITY CATEGORIES

Much has been written and said in recent years about the basis for and the adequacy of systems of stability categories represented in terms of routinely available data. Here also the time is probably ripe for a 'new look', which on the fundamental side would recognise the advances in (a) and (b) above, and on the practical side would reconsider the relative merits of the various forms of routine data.

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APPENDIX A. CROSSWIND SPREAD IN RELATION TO WIND DIRECTION FLUCTUATION

The relation between crosswind spread from a continuous point source and the crosswind component of turbulence, σ_v, has been developed on useful lines in terms of the G. I. Taylor statistical theory (Ref 6, p 185). Neglecting the variation with height of the r.m.s. and scale of the v-component, the result for travel time t or distance downwind x may be put in the forms

$$\sigma_y = \sigma_v \, t \, f_1(t/t_l) = \sigma_v (x/u_e) \, f_1(x/u_e t_l) \qquad (1)$$

where t_l is the Lagrangian integral time scale, u_e is the effective advecting speed acting on the plume of particles (which should be regarded as increasing with the vertical spread of the particles, in accordance with the increase of mean wind speed with height[7]). The function $f_1(t/t_l)$ depends on the shape of the Lagrangian autocorrelation coefficient and has the limits 1 for $t = 0$ and $(2t_l/t)^{1/2}$ for $t = \infty$. The equation may be rearranged, with $\sigma_v/\bar{u} = \sigma_\theta$ into the form

$$\sigma_y/\sigma_\theta = x \, f_1(x/u_e t_l) \qquad (2)$$

Analysis of field data in relation to expressions of the above forms has been considered by Draxler[18] and the writer[7]. It has been noted (Ref 6, p 195) that several sets of test data over relatively open sites in the U.S.A.[13] conform to the following simple relation within fairly close limits

$$\sigma_y/\sigma_\theta = x \, f(x) \qquad (3)$$

where f(s) approximates to the required limit when s = 0.1 km, and is a slowly decreasing function at longer range. The upper and lower bounds only of these σ_y data are reproduced in Fig. 1.

It has also been noted (Ref 6, p 205) that measurements of σ_y and σ_θ in two large towns in England are also related in conformity with the bounds of the U.S.A. tests. Data obtained in the St. Louis urban dispersion study[12], for a sampling time of 1 hr, have now been analysed on the same basis and the individual hourly values are plotted on Fig. 1, with discrimination according to day or night. The property σ_θ was actually observed at two levels on a television tower, and in the present analysis choice was made of the upper level (459 ft) as the more appropriate to the average magnitude of vertical spread σ_z also deduced in these tests. These data too are strikingly consistent with the bounds of the open-site data. Finally, the overall range of some data pertaining to very light winds and stable conditions, with very large values of σ_θ, obtained in field tests in Idaho, are shown.

The overall impression from these data and other data which have been reviewed previously (Ref 6, p 185) is that the crosswind spread may be usefully represented on average by the simple equation (3), with f(s) as now determined by eye from Fig. 1 taking approximate values as follows:

s (km)	0.1	0.2	0.4	1	2	4	10	>10
f(s)	0.4	0.7	0.65	0.6	0.5	0.4	0.33	$0.33(10/s)^{1/2}$

the value for $s = 10$ km being set on the reasonable assumption that the 'large s' limit, corresponding to the 'large t' limit referred to earlier, is effectively operative for such distances, at any rate in respect of the *direct* effect of the crosswind component of turbulence (see Ref. 6, p 367 for earlier discussions of this aspect).

Departures from the simple average relation are presumably a consequence partly of variations of σ_y with height, and of the product $u_z t_L$ with dispersive conditions. However, even for the individual sample data in Fig. 1 the departures are mostly within a factor of 1.5 at short range and 2.0 at long range.

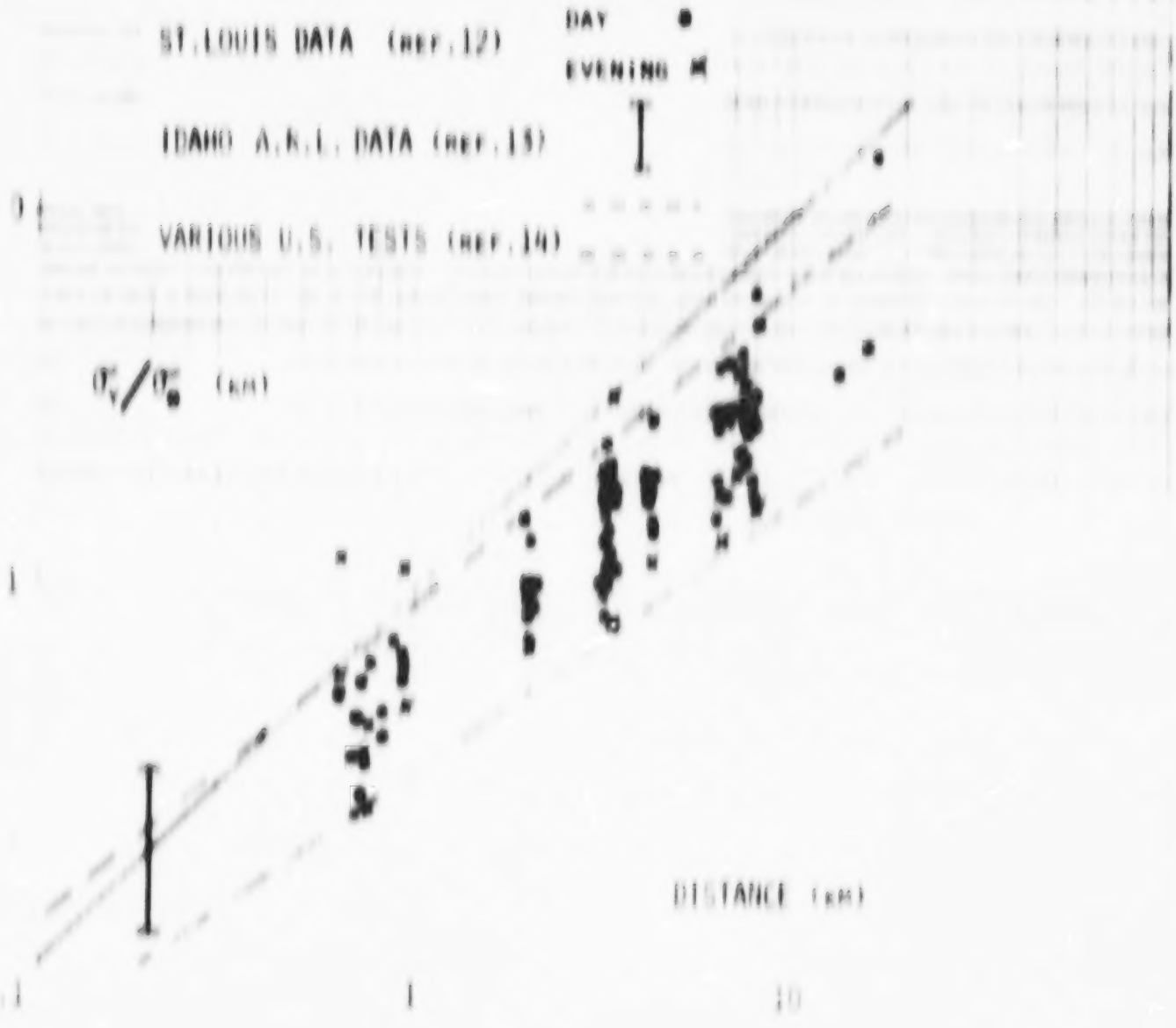

Fig. A-1. Vertical Data on Crosswind Spread Normalized to A Comparison with the Standard Deviation of Wind Direction Fluctuation

APPENDIX B. SOME CALCULATIONS OF VERTICAL SPREAD IN UNSTABLE CONDITIONS

Vertical spreads derived by F. B. Smith from numerical solutions of the two-dimensional equation of diffusion have been shown[8] to approximate to certain simple similarity relations. With $\bar{y}$ the mean vertical displacement of passive particles at distance x downwind of a continuous release at the surface, one relation is

$$d\bar{y}/dx = 0.75\,(\sigma_v/\bar{u})_z \qquad (1)$$

the subscript outside the parentheses denoting the height to which all quantities inside refer. Substituting the basic form of eddy diffusivity K adopted in Smith's solutions, i.e., $0.1\,\sigma_w\,\lambda_m$, where λ_m is the equivalent wavelength ($= \bar{u}/n_m$) for maximum value of the product of frequency (n) and spectral density $S(n)$ of the vertical component,

$$d\bar{y}/dx = 0.075\,(\sigma_w\lambda_m/\bar{u})_z \qquad (2)$$

This relation may be used to obtain new $\bar{y}/x$ or σ_y/x curves from new σ_w and λ_m data in convective conditions, recently made available from the 'Minnesota' boundary layer experiment (Ref 9).

For convenience we may integrate (2) in separate sections of the $\bar{y}$ range, for which simple mathematical forms may be prescribed as approximations to the observed profiles of σ_w and λ_m contained in Figs. 5 and 14 of Ref 9:

Section A

$$h > x > 0.1h', \quad \sigma_w/w_* = 1.3\,(z/h')^{1/3} \qquad (3)$$

$$\lambda_m = 5.9\,z \qquad (4)$$

where L is the Monin-Obukhov length, and h' the mixing depth

Section B

$$0.1h' < z < h';\ \sigma_w/w_* = 0.55 \tag{5}$$

$$\ell_m/h' = 1.7(z/h')^{0.4} \tag{6}$$

where w_* is Deardorff's scaling velocity and the symbol h' is used here for mixing depth to avoid confusion with the use of z_i below. The lower limit $-h$ in Section A is adopted as appropriate to the beginning of the fully-convective regime, in which the properties are independent of u_*. For smaller z, (in which u_* (or z_0) are relevant, F. B. Smith's original solutions are retained as adequately consistent with Monin-Obukhov similarity.

Integrating Eq. (2)

$$s(\bar{T}_2) - s(\bar{T}_1) = (1/0.075)\int_{\bar{T}_1}^{\bar{T}_2} (d\bar{u}/u_*\ell_m)\, dz \tag{7}$$

For Section A we take $\bar{u}$ a function of height, and for convenience use a simple power-law approximation to the profile in F. B. Smith's solution in unstable conditions, assuming this to apply up to $0.1h'$, i.e.,

$$\bar{u}(z)/u_1 = (10z/h')^{0.075} \tag{8}$$

with u_1 the effectively constant speed holding above $0.1h'$ and hence throughout Section B (this effective constancy was observed in the 'Minnesota' study). Substituting the appropriate profiles of σ_w and ℓ_m in Eq. (7) the resulting expressions are

Section A (Eq. (3) and (4) applicable)

$$s(\bar{Y}) - s(-H) = \left[2.78u_* h'^{0.25}/w_s\right]\left[\bar{Y}^{0.74} - (-H)^{0.74}\right] \qquad (9)$$

Section B (Eq. (5) and (6) applicable)

$$s(\bar{Y}) - s(0.1h') = \left[0.9u_*/w_s h'^{0.6}\right]\left[\bar{Y}^{1.6} - (0.1h')^{1.6}\right] \qquad (10)$$

Note that if, as usual, wind speed is specified at a given low reference level (say, 10m), then for such a given wind speed u_* is proportional to $h'^{0.075}$ (in accordance with Eq. (8), and remembering that w_s is proportional to $h'^{1/2}$ it follows that, as required by definition, the relation between $\bar{Y}$ and s in Eq. (9) is independent of h'. Note also that if the variation of wind with height is disregarded in Section A Eq. (9) reduces to

$$s(\bar{Y}) - s(-H) = \left[2.61 \quad ih'^{1/2}/w_s\right]\left[\bar{Y}^{1/3} - (-H)^{1/3}\right] \qquad (11)$$

which for large values of $\bar{Y}/H$ is of the same form as Eq. 3 in the main text.

In the trial calculations carried out here, Eq. (9) and (10) have been used in the slightly rearranged forms

$$s(\bar{Y}) - s(-H) = f_A\left[(\bar{Y}/-H)^{0.74} - 1\right] \qquad (12)$$

$$\text{with } f_A = 2.78u_* h'^{0.25}(-H)^{0.74}/w_s$$

$$s(\bar{Y}) - s(0.1h') = f_B\left[(\bar{Y}/0.1h')^{1.6} - 1\right] \qquad (13)$$

$$\text{with } f_B = 0.9u_* h'(0.1)^{1.6}/w_s$$

the $\bar{Y}$,s data calculated from these equations were ultimately converted to σ_w,s data by assuming $\sigma_z = 1.25\bar{Y}$, which applies for a Gaussian distribution.

Calculations have also been carried out from the free-convection similarity law in Eq. (3) of Section 3(c), converting from time to distance by assuming wind speed constant with height and equal to u_c as defined above, and adopting the value 0.5 for the coefficient $\underline{a}$. This value was given by Isserdorff and Willis[11] on the basis of their laboratory data and was shown by the writer[8] to be consistent with the similarity form given at the beginning of this appendix. The complete expression is

$$\sigma_z(x) = a(x/u_c)^{3/2}(g/cc_p T)^{1/2} H_0^{1/2} \qquad (14)$$

The data used in the calculations are summarized in Table I and the results are shown graphically in Figs. 1, 2 and 3, which also include the appropriate PG curves and curves derived from F. B. Smith's nomogram system.

Before listing the conclusions it should be noted that somewhat arbitrary choices have been made for some of the parameter values, notably

(i) the choice of 1 hr as the $\bar{Y}$ limit up to which the F. B. Smith calculations are restrained.

(ii) the choice of mixing depths, which are almost certainly larger than normally expected.

(iii) the assumption of 15 cal/cm^2 as the man-made contribution to H_0.

It seems unlikely, however, that any plausible deviations from these choices would basically change the implications listed below. Note in particular that the largest magnitudes of σ_z are essentially set by $H_0^{1/2}$, so a substantial latitude in the value adopted for H_0 would have a minor effect.

The conclusions may now be summarized as follows:

1. For a smooth surface, gradient-transfer calculations using the latest data available on the intensity and scale of the w-component in convective conditions give σ_z values consistent with the short-range (0.5 km) section of the PG 'A' curve, which in turn is consistent with available observations for a surface release. With increasing distance the PG values (which are essentially subjective extrapolations) exceed the gradient-transfer calculations by a progressively increasing amount, possibly reflecting a 'convective enhancement' of the vertical spread which is not representable by a gradient-transfer mechanism.

2. A 'free-convection' law provides σ_z or $\bar{T}$ results which are not inconsistent with the gradient-transfer interpretation, over the height-range for which the σ_w data follow a 'free-convection' similarity law. If the 'free-convection' law for σ_z is taken to be applicable to much greater heights (as implied by Deardorff

and Willis' (Ref. 11) tank data) the calculations approach closer to the PG curve, but are still on the low side of the curve.

3. Over a rough (urban) surface, when even with large heat flux the magnitude of -L is relatively large and accordingly when the gradient-transfer assumption should be more realistic over a greater height range, the gradient-transfer calculations still fall below those inferred from ground-level concentrations of tracer in the St. Louis study. The 'free-convection' law, which by the same argument about -L should be less valid compared with a smooth surface and small |L| does give a closer approach to the St. Louis data, though with a more rapid variation of σ_y with x than was inferred from the concentration data.

4. The implications regarding Deardorff's laboratory law for the 'mixed layer' are at present ambiguous. Over the bottom half of this layer the laboratory data on σ_y are consistent with the 'free-convection' law, which on first principles might be expected to fail above z/h' = 0.1, and which in respect of σ_y apparently does fail in both laboratory and atmospheric data. It is understood, however, that a more recent statement of the laboratory data on σ_y is in course of publication, and it is clearly desirable that this should be fully considered in the present context before drawing final conclusions.

TABLE B-1. DATA USED IN SPECIMEN CALCULATIONS OF EQ. (12), (13) and (14)

Case	1	2	3
z_0 cm	3	100	100
H_0 mw/cm^2	20	40	80
$\bar{u}$ (10m) m/sec	2	4	6
approx z_i m[i]	10	250	600
assumed h' km[ii]	1	5	4
u_* (from Eq. 8) m/sec	0.58	0.96	0.45
w_s m/sec	0.75	3.80	0.90
f_A in Eq. 12 m	115	2140	--
f_B in Eq. 13 m	630	1575	1490
Stability category[iii]	A	A	B
'Workbook' Stability category[iv]		B	C

Notes: (i) Estimated from the nomogram on p 338 of Ref. 6

(ii) In cases 2 and 3 these values of h' have been adopted as necessarily larger than the derived z_i values in the St. Louis urban study.

(iii) In accordance with F. B. Smith's nomogram, p 174 of Ref. 6 and the magnitudes of H_0 and $\bar{u}$(10m)

(iv) Obtained as in (iii) but for (H_0 - 15) mw/cm^2 on the supposition that 15 mw/cm^2 is the typical man-made contribution to H_0 in a modern city. The 'Workbook' stability category is then the category which would be assessed in the normal way in the Workbook.

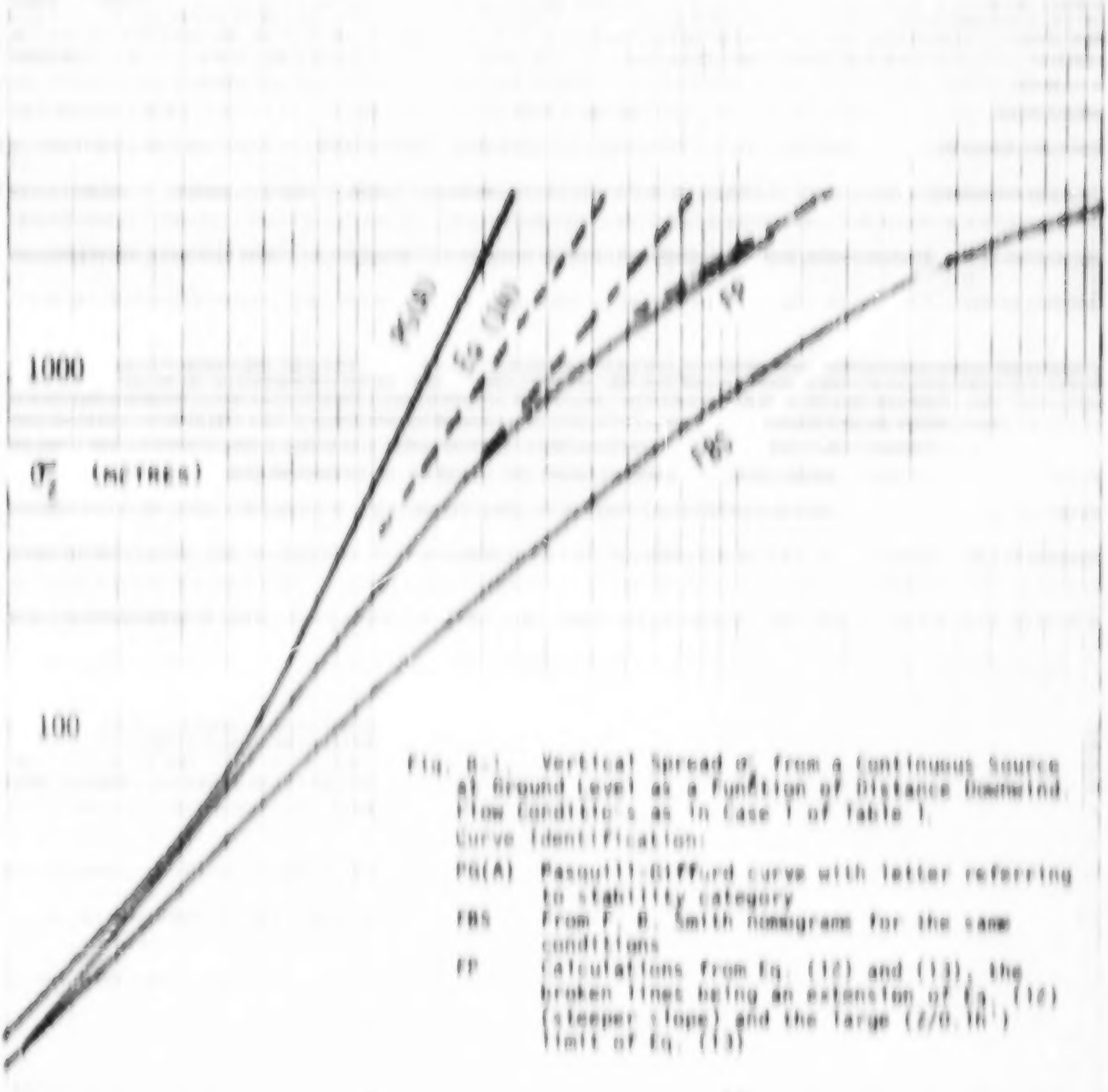

Fig. B-1. Vertical Spread σ_z from a Continuous Source at Ground Level as a Function of Distance Downwind. Flow Conditions as in Case 1 of Table 1.

Curve Identification:

PG(A)	Pasquill-Gifford curve with letter referring to stability category
FBS	From F. B. Smith nomograms for the same conditions
FP	Calculations from Eq. (12) and (13), the broken lines being an extension of Eq. (12) (steeper slope) and the large ($z/0.16^{1}$) limit of Eq. (13)

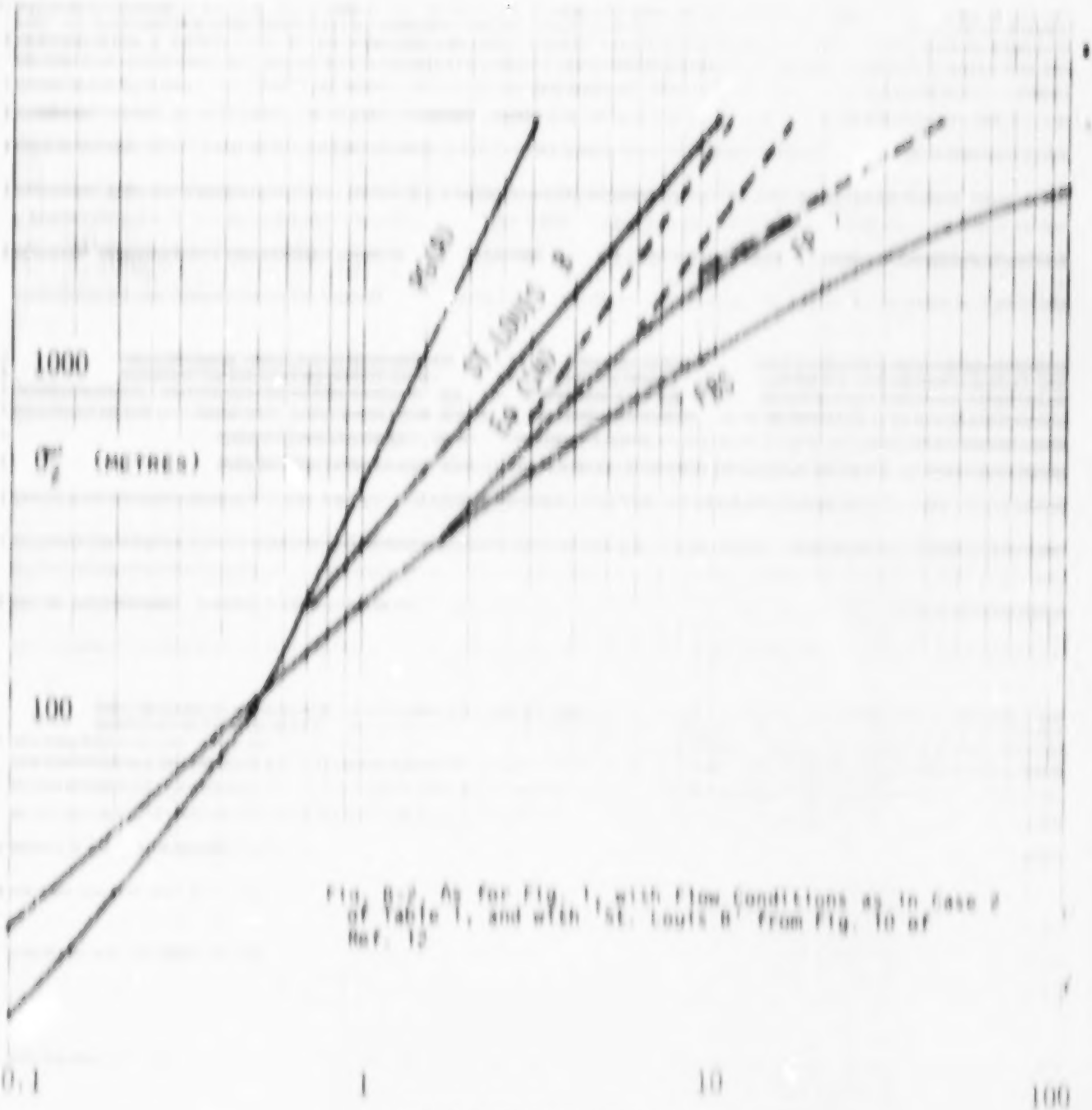

Fig. B.2 As for Fig. 1 with flow conditions as in Case 2 of Table 1, and with 'St. Louis B' from Fig. 10 of Ref. 12

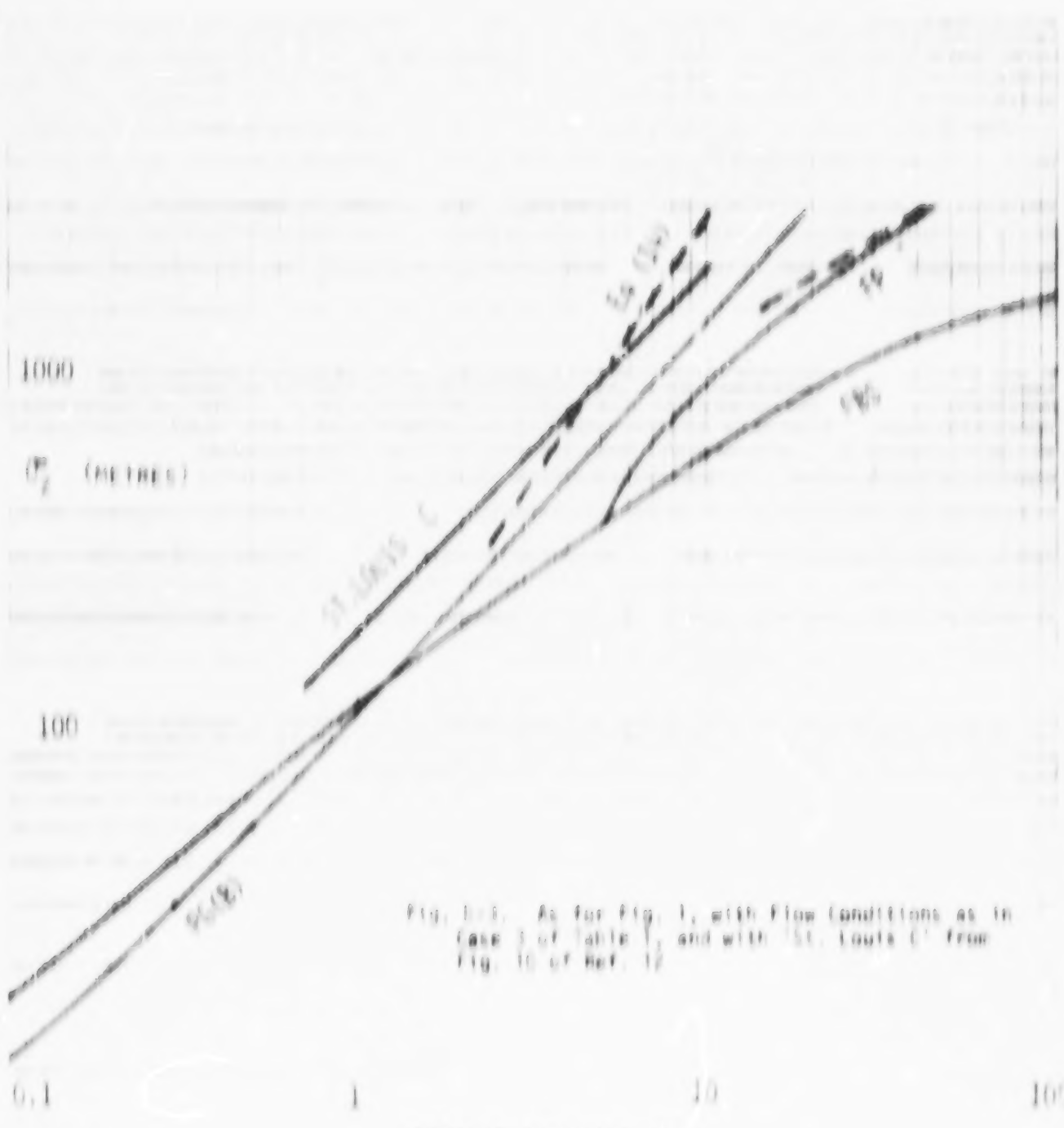

Fig. 1.5. As for Fig. 1, with flow conditions as in case 1 of Table 1, and with St. Louis C from Fig. 10 of Ref. 12